

# Diagnostic of brownstock washing using basic filtration parameters

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Bamboo Award: Best Technical article in engineering & maintenance at the ABTPC-PI 2005 Congress

**Keywords:** Washing, Filtration, Brownstock Washing, Rotary Drum Vacuum Filter

## ABSTRACT

It was showed the modeling and calculation of washing operation by a rotary drum vacuum filter. A mathematical model was elaborated based on the fundamental theory of filtration at constant pressure and from empiric parameters, determined experimentally. It was applied the technique of "System Engineering Analysis" to obtain the values of the operational variables in all section that composes the filter, allowing the diagnosis of the filter operation. A parametric study is done to obtain the washing efficiency and the filtration capacity.

## INTRODUCTION

Brownstock washing is the operation where organic material and dissolved inorganic chemical substances are separated from cellulosic fibers. One of the purposes of the washing is to remove the residual liquor that might

contaminate the pulp during the subsequent stages of the process. Another objective is to recover valuable dissolved substances, such as the organic material in the black liquor that is used as fuel in the recovery boiler and inorganic chemical substances for the regeneration of the white liquor for the cooking. An efficient washing requires the control of the volume of washing fluid that is added to the system. Using large amounts of washing fluid we obtain a cleaner pulp, but an efficient operation of the recovery system requires minimum dilution of the black liquor, in order to reduce the consumption of energy during the process of evaporation. On the other hand, in case of insufficient washing, there is an excessive loss of black liquor, which affects the thermal balance of the line and of the chemical products in the recovery section, as well as leads to a greater consumption of oxidizing agents during bleaching, which generates a greater load of polluting materials. Therefore, the

performance of an efficient washing contributes significantly towards a better energy balance, a lower consumption of water and chemical products, and a reduction in the generation of polluting effluents.

One of the first equipment used in continuous operations to wash the brownstock was the rotary drum vacuum filter, which is still used nowadays. The filter consists in a perforated drum that is covered with a filtering medium, usually a synthetic or metallic screen mesh. During the operation of the filter, the drum is partially immersed in a vat that is fed with a diluted suspension. The vacuum applied through the drum of the filter extracts part of the black liquor from the suspension of brownstock, forming a cake at the surface of the filtering medium. As the drum rotates, spray showers spread the washing fluid on top of the pulp cake, displacing the liquor that is present in the cake for a liquor with a lower concentration of solids. The cake is then separated from the surface of the filtering medium with the interruption of the vacuum. A washing

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system uses a number of filters, identical or different, in series, with the filtrate flowing countercurrent with respect to the brownstock that is being washed.

In this paper the rotary drum vacuum filter was split into unit operations, and for each one it was prepared a mathematical model based on the fundamental theory of filtration at constant pressure. It was developed an algorithm from the "System Engineering Analysis" methodology - as per Rudd and Watson (1968), Barton (1995) and Barton (1998) - in order to have available the values for the operating variables in all inlet and outlet sections of the unit operations that make up the rotary drum vacuum filter, allowing in this description a diagnostic of the operation.

The results that are obtained from the calculation procedure provide information that is complementary to those that are available from the control panel and/or from the usual chemical analyses that are made for the washing filter system. It is thus an important tool in the diagnostic and the management of the washing operation of the brownstock.

### ANALYSIS OF THE ROTARY DRUM VACUUM WASHING FILTER

The rotary drum vacuum filter may be split into modules of unit operations. According to Edwards *et al.* (1986) such procedure is important for a better understanding of the washing process. In the scheme that is illustrated in Figure 1 the feed suspension (1) is diluted in a stirred tank, which feeds the filtration vat with a suspension with a given consistency. The diluted pulp (2) is transferred to the filtration vat where the water is drained through filtration for the cake formation in a suction drum. When the region of the drum enters the washing section (4) the washing fluid (7) displaces the fluid that is present in the cake (8) and the cake (6) undergoes a last draining where it is thickened (9) to be discharged to the

next equipment in the line of operation. In the present study the filtrates from the stages of filtration (5), washing (8) and draining (10) are mixed in a tank from which a fraction of the flowrate (11) is split, returning one part (3) to the dilution module, while the other part (12) returns to the other operations of the line.

### MATHEMATICAL MODELING OF THE ROTARY DRUM VACUUM FILTER

A mathematical model was developed for each one of the modules in Figure 1 from specific variables.

Each stream that is specified in Figure 1 may be defined by the following parameters: ( $M_{Pi}$ ) mass flowrate of the cellulose pulp; ( $M_{Wi}$ ) mass flowrate of water; ( $S_{SPi}$ ) consistency of the brownstock pulp suspension; ( $W_{Pi}$ ) ratio between the mass of water and the mass of pulp; ( $X_{Si}$ ) ratio between the mass of (pulp free) soluble solids and the mass of water.

$W_{Pi}$  may be given, in a generic stream ( $i$ ) by:

$$W_{Pi} = \frac{1 - S_{SPi}}{[S_{SPi}(1 + X_{Si}) - X_{Si}]} \quad (1)$$

### MASS BALANCE

In steady state, the mass balances for each one of the individual modules in Figure 1 are described by the equations given below.

#### Module of dilution:

- Mass balance for the cellulose pulp:

$$M_{P2} = f_1 M_{P1} \quad (2)$$

- Mass balance for the water:

$$M_{P1} W_{P1} + M_{W3} = M_{P2} W_{P2} \quad (3)$$

- Mass balance for the soluble solids:

$$M_{P1} W_{P1} X_{S1} + M_{W3} X_{S3} = M_{P2} W_{P2} X_{S2} \quad (4)$$

#### Module of filtration:

- Mass balance for the cellulose pulp:

$$M_{P4} = f_2 M_{P2} \quad (5)$$

- Mass balance for the water:

$$M_{P2} W_{P2} = M_{P4} W_{P4} + M_{W5} \quad (6)$$

- Mass balance for the soluble solids:

$$M_{P2} W_{P2} X_{S2} = M_{P4} W_{P4} X_{S4} + M_{W5} X_{S5} \quad (7)$$

#### Module of washing:

- Mass balance for the cellulose pulp:

$$M_{P6} = f_3 M_{P4} \quad (8)$$

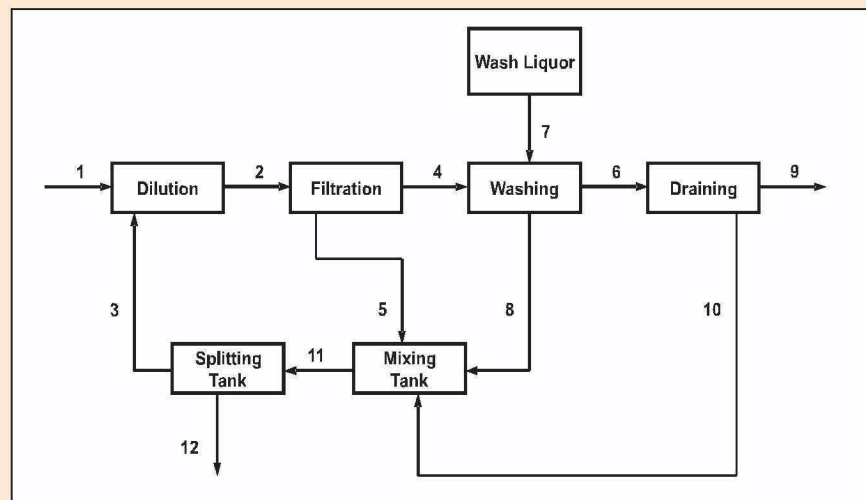


Figure 1. Flow diagram for rotary drum vacuum filter

• Mass balance for the water:

$$M_{p4}W_{p4} + M_{w7} = M_{p6}W_{p6} + M_{w8} \quad (9)$$

• Mass balance for the soluble solids:

$$M_{p4}W_{p4}X_{s4} + M_{w7}X_{s7} = M_{p6}W_{p6}X_{s6} + M_{w8}X_{s8} \quad (10)$$

#### Module of draining:

• Mass balance for the cellulose pulp:

$$M_{p9} = f_4 M_{p6} \quad (11)$$

• Mass balance for the water:

$$M_{p6}W_{p6} = M_{p9}W_{p9} + M_{w10} \quad (12)$$

• Mass balance for the soluble solids:

$$M_{p6}W_{p6}X_{s6} = M_{p9}W_{p9}X_{s9} + M_{w10}X_{s10} \quad (13)$$

#### Module of the mixing tank:

• Mass balance for the water:

$$M_{w5} + M_{w8} + M_{w10} = M_{w11} \quad (14)$$

• Mass balance for the soluble solids:

$$M_{w5}X_{s5} + M_{w8}X_{s8} + M_{w10}X_{s10} = M_{w11}X_{s11} \quad (15)$$

#### Module of the splitting tank:

• Mass balance for the water:

$$M_{w3} + M_{w12} = M_{w11} \quad (16)$$

• Mass balance for the soluble solids:

$$M_{w3}X_{s3} + M_{w12}X_{s12} = M_{w11}X_{s11} \quad (17)$$

### CONSTANT PRESSURE FILTRATION

For filtrations at a constant pressure, the filtration time ( $t_f$ ) is given by equation (18).

$$t_f = \frac{K_C V_F^2}{2\Delta P_F} + \frac{K_{MF} V_F}{\Delta P_F} \quad (18)$$

where:

$$K_C = \frac{\alpha S_{SP2} \rho_s \mu_s}{(1 - S_{SP2} F_{WS}) A_F^2} \quad (19)$$

and

$$K_{MF} = \frac{R_{MF} \mu_s}{A_F} \quad (20)$$

The filtration time ( $t_f$ ) is given by the division of the angle of the filtration section ( $\theta_f$ ) by the angular velocity of the filter drum ( $\omega$ ).

$$t_f = \frac{\theta_f}{\omega} \quad (21)$$

Equations (22) and (23) relate the variables of the module of filtration with the variables for the flow inside the washing filter.

$$M_{w5}(1 + X_{s5}) = \rho_s \frac{V_F}{t_f} \quad (22)$$

$$M_{p4}(1 + W_{p4}X_{s4}) = \frac{S_{SP2} \rho_s}{(1 - S_{SP2} F_{WS})} \frac{V_F}{t_f} \quad (23)$$

The fraction of water per cellulose in the cake ( $W_{p4}$ ) may be given as a function of the mass relation between the wet cake and the dry cake ( $F_{WS}$ ):

$$W_{p4} = \frac{F_{WS} - 1}{(1 - X_{s4}) - F_{WS} X_{s4}} \quad (24)$$

### WASHING

The washing of a cake after the stage of filtration takes place by displacement and diffusion. To calculate the washing time ( $t_L$ ) it is assumed that the conditions of the flow are the same as the ones that existed at the end of the stage of filtration, that is, the structure of the cake is not affected when the washing liquid displaces the liquid that is present in the cake from the stage of filtration.

Like in equation (21), the washing time ( $t_L$ ) is obtained dividing the angle of the washing section ( $\theta_L$ ) by the angular velocity of the filter drum ( $\omega$ ).

$$t_L = \frac{\theta_L}{\omega} \quad (25)$$

The washing area ( $A_L$ ) is related to the filtration area ( $A_F$ ) and the washing ( $\theta_L$ ) and filtration angles ( $\theta_f$ ):

$$A_L = A_F \left( \frac{\theta_L}{\theta_f} \right) \quad (26)$$

In the case of rotary drum vacuum filters, the washing area ( $A_L$ ) is different from the filtration area ( $A_F$ ), and thus we have to correct the characteristic coefficient of the cake ( $K_{CL}$ ) and the characteristic coefficient of the filtering medium ( $K_{MFL}$ ) for the washing section, equations (28) and (29), respectively. In filters where the washing fluid flow in one single direction and which are operating at a constant pressure, the washing time ( $t_L$ ) can be estimated by:

$$t_L = \frac{(K_{CL} V_F + K_{MFL}) V_L}{\Delta P_L} \quad (27)$$

where:

$$K_{CL} = K_C \left( \frac{\theta_L}{\theta_f} \right) \quad (28)$$

and

$$K_{MFL} = K_{MF} \left( \frac{\theta_L}{\theta_f} \right) \quad (29)$$

Equation (30) relates the time ( $t_L$ ) and the volume ( $V_L$ ) with the mass balance for the washing stage.

$$M_{w8}(1 + X_{s8}) = \rho_s \frac{V_L}{t_L} \quad (30)$$

### WASHING EFFICIENCY

According to Gullichsen (2000) and Casey (1980), two parameters determine the performance of the washing system: the dilution factor ( $FD$ ) and the displacement ratio ( $RD$ ). The amount of water that is used for washing the pulp is usually given by the dilution factor ( $FD$ ), which is defined as the amount of washing water that exceeds that which is ideally required for a total displacement. A negative dilution factor represents the case where less washing water is added to the system than the amount of water

that exits the washer. Equation (31) provides the mathematical expression for the dilution factor ( $FD$ ):

$$FD = \left( \frac{M_{w7}}{M_{p2}} \right) - W_{p9} \quad (31)$$

The efficiency of a single washing stage in the removal of the solids in the pulp may be given in terms of the displacement ratio ( $RD$ ), which is defined as the relation between the reduction of solids in one single stage and the maximum possible reduction in that stage. Equation (32) gives the displacement ratio ( $RD$ ) as a function of the fractions of soluble solids ( $X_{Si}$ ):

$$RD = \left( \frac{X_{S2} - X_{S9}}{X_{S2} - X_{S7}} \right) \quad (32)$$

The overall washing system efficiency ( $Y$ ) is also defined according to the mass balance for soluble solids:

$$Y = \frac{M_{w12}X_{S12} - M_{w7}X_{S7}}{M_{p1}W_{p1}X_{S1}} \quad (33)$$

Nordén (1973) *apud* Rogers *et al.* (1995) proposes another method for measuring the washing efficiency of the pulp. The Nordén number ( $E$ ), equation (34), is defined as the number of ideal washing stages per countercurrent extraction, needed for obtaining the same performance as a given washer with the same discharge consistency and dilution factor. As the discharge consistency is variable, this method can not be used directly for the comparison of washers with different discharge consistencies.

$$E = \frac{\log \left( \frac{W_{p2} \left( \frac{X_{S2} - X_{S11}}{X_{S9} - X_{S7}} \right)}{W_{p9}} \right)}{\log \left( \frac{M_{w7}}{M_{p9}W_{p9}} \right)} \quad (34)$$

### LOCAL WASHING EFFICIENCY

The local efficiency ( $Y_L$ ) for the washing module may be given by:

$$Y_L = \frac{M_{p4}W_{p4}X_{S4} + M_{w7}X_{S7} - M_{p6}W_{p6}X_{S6}}{M_{p4}W_{p4}X_{S4} + M_{w7}X_{S7}} \quad (35)$$

Substituting equation (10) into equation (35) we have:

$$Y_L = \frac{M_{w8}X_{S8}}{M_{p6}W_{p6}X_{S6} + M_{w8}X_{S8}} \quad (36)$$

Equation (37) determines the lower limit for the local washing efficiency ( $Y_{Li}$ ), in which it is assumed the existence of a stage of equilibrium; in this case we have the perfect mixing of the liquor that is present in the cake with the liquor that is applied by the spray showers, hence  $X_{S6} = X_{S8}$ .

$$Y_{Li} = \frac{M_{w8}}{M_{p6}W_{p6} + M_{w8}} \quad (37)$$

Equation (38) gives the upper limit for the local washing efficiency ( $Y_{Ls}$ ), in this case, we assume that all the liquor that is present in the cake is displaced by the liquor that is applied by the spray showers, where the flow of the liquor is a plug flow, that is,  $X_{S6} = X_{S7}$ .

$$Y_{Ls} = 1 - \frac{M_{p6}W_{p6}X_{S7}}{M_{p4}W_{p4}X_{S4} + M_{w7}X_{S7}} \quad (38)$$

Therefore, the local washing efficiency ( $Y_L$ ) may be given in terms of the upper limit ( $Y_{Ls}$ ) and the lower limit of the washing ( $Y_{Li}$ ), through the equation:

$$Y_L = Y_{Ls}x_f + (1 - x_f)Y_{Li} \quad (39)$$

Where:  $x_f \rightarrow 0$  implies a "perfect mixing" while  $x_f \rightarrow 1$  implies a "plug flow".

### HYPOTHESES OF THE MODEL

The following hypotheses were adopted for the solution of the system of equations:

1. There is no loss of cellulose pulp in the filter, therefore,  $f_1 = f_2 = f_3 = f_4 = 1$
2. The head loss during the process of washing ( $\Delta P_L$ ) is equal to

the head loss in the process of filtration ( $\Delta P_L$ ):

$$\Delta P_L = \Delta P_F \quad (40)$$

3. The amount of soluble solids in stream 4 ( $M_{p4}W_{p4}X_{S4}$ ) is negligible when compared to the amount of cellulose pulp ( $M_{p4}$ ), thus equation (23) may be simplified:

$$M_{p4} = \frac{S_{SP2}\rho_s}{(1 - S_{SP2}F_{WS})} \frac{V_F}{t_F} \quad (41)$$

4. The amount of soluble solids ( $M_{wi}X_{Si}$ ) in streams 5 and 8 is negligible when compared to the amount of water ( $M_{wi}$ ), therefore equations (22) and (30) may be simplified to:

$$M_{w5} = \rho_s \frac{V_F}{t_F} \quad (42)$$

$$M_{w8} = \rho_s \frac{V_L}{t_L} \quad (43)$$

5. The concentrations of soluble solids in the inlet and outlet streams of the stages of filtration, draining, and the splitting tank are identical, therefore the fraction  $X_{Si}$  in these stages is given, respectively, in the following way:

$$\Delta P_L = \Delta P_F \quad (44)$$

$$X_{S5} = X_{S2} \quad (45)$$

$$X_{S9} = X_{S6} \quad (46)$$

$$X_{S10} = X_{S6} \quad (47)$$

$$X_{S11} = X_{S3} \quad (48)$$

$$X_{S12} = X_{S3} \quad (49)$$

6. The value of the fraction of solids  $X_{Si}$  is very small when compared to the fraction  $W_{Pi}$ , hence equations (1) and (24) may be simplified to give:

$$W_{Pi} = \frac{1 - S_{SPi}}{S_{SPi}} \quad (50)$$

$$W_{P4} = F_{WS} - 1 \quad (51)$$

The value for the specific resistance of the cake ( $\alpha$ ) is determined experimentally using the "Leaf Test" procedure [Perry (1997)]. For cellulose cakes, compressible, this value may be estimated, for different filtration pressures, using equation proposed by Reynol (2005).

$$\alpha = 7,76 \cdot 10^5 \Delta P_C^{0,77} \quad (52)$$

### "SYSTEM ENGINEERING ANALYSIS"

If a system of equations is made of  $\lambda$  non linear equations and  $\lambda$  variables, this system has one single solution. However, the number of ways in which the equations may be ordered is equal to  $(\lambda!)^2$ . Therefore, if  $\lambda$  is a number that is greater than or equal to 3, the structure of the calculation algorithm can not be made by inspection. Hence, a more systematic method is to be sought to accomplish this.

For such, the algorithm described by Rudd and Watson (1968), Barton (1995) and Barton (1998) may be used, where a set of equations and variables is obtained, which will be calculated through the employment of these equations. Such procedure was detailed by Reynol *et al.* (2005) and Reynol (2005).

### ALGORITHM OF THE CALCULATION PROCEDURE

The set of equations for the filter exhibit a number of variables that is

equal to  $\sigma = 61$  and a number of equations that is equal to  $\lambda = 43$ , therefore the degree of freedom of the system is of  $GL = \sigma - \lambda = 18$ . As such, it must be assumed that the following design variables are known:

#### For the operating conditions:

- Dilution factor (FD).
- Consistency of the brownstock pulp at the inlet of the filter ( $S_{SP1}$ ).
- Consistency of the brownstock pulp at the outlet of the filter ( $S_{SP9}$ ).
- Fraction of soluble solids at the inlet of the filter ( $X_{S1}$ ).
- Fraction of soluble solids applied by the spray shower ( $X_{S7}$ ).
- Angular velocity of the filter cylinder:  $\omega$ .

#### For the module of filtration:

- Cake humidity:  $F_{WS}$ .
- Head loss during the process of filtration:  $\Delta P_F$ .
- Resistance of the filtering medium:  $R_{MF}$ .
- Consistency of the brownstock suspension at the vat of the filter ( $S_{SP2}$ ).
- Specific resistance of the cake:  $\alpha$ .
- Dynamic viscosity of the filtrate in exit stream no. 5:  $\mu_5$ .
- Density of the filtrate in exit stream no. 5:  $\rho_5$ .

#### For the module of washing:

- Density of the filtrate in exit stream no. 8:  $\rho_8$ .

#### For the characteristics of the filter:

- Filtration area:  $A_F$ .
- Angle of the filtration section:  $\theta_F$ .
- Angle of the washing section:  $\theta_L$ .

#### Parameter of the washing mechanism:

- Factor  $x_{f..}$ .

The algorithm of the calculation procedure is fed with the data that is given above, that is, with the design

variables. The calculation sequence may be summarized as below:

1.  $t_F$  is calculated using equation (21);
2.  $t_L$  is calculated using equation (25);
3.  $K_C$  is calculated using equation (19);
4.  $K_{MF}$  is calculated using equation (20);
5.  $A_L$  is calculated using equation (26);
6. With  $K_C$  we calculate  $K_{CL}$  using equation (28);
7. With  $K_{MF}$  we calculate  $K_{MFL}$  using equation (29);
8.  $\Delta P_L$  is calculated using equation (40);
9. With  $K_C$ ,  $K_{MF}$  and  $t_F$  we obtain  $V_F$  using equation (18);
10. With  $K_{CL}$ ,  $K_{MFL}$ ,  $t_F$ ,  $\Delta P_L$  and  $V_F$  we obtain  $V_L$  using equation (27);
11. With  $V_F$  and  $t_F$  we calculate  $M_{P4}$  using equation (41);
12.  $M_{P2}$  with  $M_{P4}$  using equation (2);
13.  $M_{P1}$  with  $M_{P2}$  using equation (5);
14.  $M_{P6}$  with  $M_{P4}$  using equation (8);
15.  $M_{P9}$  with  $M_{P6}$  using equation (11);
16. With  $V_F$  and  $t_F$  we calculate  $M_{W5}$  using equation (42);
17. With  $V_L$  and  $t_L$  we calculate  $M_{W8}$  using equation (43);
18.  $W_{P1}$  is calculated using equation (50);
19.  $W_{P2}$  is calculated using equation (50);
20.  $W_{P9}$  is calculated using equation (50);
21.  $W_{P4}$  is calculated using equation (51);
22.  $M_{W7}$  with  $M_{P2}$  and  $W_{P9}$  using equation (31);
23. With  $M_{P4}$ ,  $W_{P4}$ ,  $M_{W7}$ ,  $M_{P6}$  and  $M_{W8}$  we calculate  $W_{P6}$  using equation (9);
24. With  $M_{P6}$ ,  $W_{P6}$ ,  $M_{P9}$  and  $W_{P9}$  we calculate  $M_{W10}$  using equation (12);
25. With  $M_{W5}$ ,  $M_{W8}$  and  $M_{W10}$  we calculate  $M_{W11}$  using equation (14);
26. With  $M_{P1}$ ,  $W_{P1}$ ,  $M_{P2}$  and  $W_{P2}$  we calculate  $M_{W3}$  using equation (3);
27. With  $M_{W11}$  and  $M_{W3}$  we calculate  $M_{W12}$  using equation (16);
28. We assume the initial value for  $X_{S4}$ ;
29. We assume the initial value for  $X_{S8}$ ;
30. Start of the loop  $X_{S8}$ ;
31. Start of the loop  $X_{S4}$ ;
32. With  $M_{P6}$ ,  $W_{P6}$  and  $M_{W8}$  we calculate  $Y_{L1}$  using equation (37);
33. With  $M_{P4}$ ,  $W_{P4}$ ,  $X_{S4}$ ,  $M_{P6}$ ,  $W_{P6}$ ,  $M_{W7}$  we calculate  $Y_{L8}$  using equation (38);

34. With  $Y_{L1}$  and  $Y_{L8}$  we calculate  $Y_L$  using equation (39);

35. With  $M_{p,p}$ ,  $W_{p,p}$ ,  $X_{s,p}$ ,  $M_{p,p}$ ,  $W_{p,p}$ ,  $M_{w7}$  and  $Y_L$  we calculate  $X_{S6}$  using equation (35);

36. Calculate  $X_{S9}$  with  $X_{S6}$  using equation (46);

37. Calculate  $X_{S10}$  with  $X_{S6}$  using equation (47);

38. With  $M_{p,p}$ ,  $W_{p,p}$ ,  $M_{p,p}$ ,  $W_{p,p}$ ,  $X_{S6}$ ,  $M_{w7}$ ,  $M_{w8}$  and  $X_{S8}$  we calculate  $X_{S4C}$  using equation (10);

39. Using the value of  $X_{S4}$  and  $X_{S4C}$  to calculate a new value of  $X_{S4}$  and returning to step 31 until we reach convergence;

40. Calculate  $X_{S2}$  with  $X_{S4}$  using equation (44);

41. Calculate  $X_{S5}$  with  $X_{S2}$  using equation (45);

42. With  $M_{p1}$ ,  $W_{p1}$ ,  $M_{p2}$ ,  $W_{p2}$ ,  $X_{S2}$  and  $M_{w3}$  we calculate  $X_{S3}$  using equation (4);

43. Calculate  $X_{S11}$  with  $X_{S3}$  using equation (48);

44. Calculate  $X_{S12}$  with  $X_{S3}$  using equation (49);

45. Calculate  $X_{S8C}$  with  $M_{w3}$ ,  $X_{S3}$ ,  $M_{w8}$ ,  $M_{w10}$ ,  $X_{S10}$ ,  $M_{w11}$  and  $X_{S11}$  using equation (15);

46. Using the value of  $X_{S8}$  and  $X_{S8C}$  to calculate a new value of  $X_{S8}$  and returning to step 30 until we reach convergence;

47. Calculate  $RD$  with  $X_{S2}$  and  $X_{S9}$  using equation (32);

48. Calculate  $Y$  with  $M_{p1}$ ,  $W_{p1}$ ,  $M_{w7}$ ,  $M_{w12}$  and  $X_{S12}$  using equation (33);

49. Calculate  $E$  with  $M_{p,p}$ ,  $W_{p,p}$ ,  $X_{S2}$ ,  $X_{S9}$ ,  $X_{S11}$ ,  $M_{w7}$  and  $X_{S9}$  using equation (34).

### INITIAL VALUES FOR THE CALCULATION PROCEDURE

In order to evaluate the model that was developed for the diagnostic of the operation of the washing of the brownstock, the conditions presented below were adopted, which are representative of the conditions of an industrial brownstock pulp washing filter.

#### For the operating conditions:

$FD = 2$  to  $6$ ;

$S_{SP1} = 0.12$  kg of dry solids / kg of suspension or 12%;

$S_{SP9} = 0.12$  kg of dry solids / kg of suspension or 12%;

$X_{S1} = 0.005$  kg of soluble solids / kg of liquor;

$X_{S7} = 0.0005$  kg of soluble solids / kg of liquor;

$\omega = 0.1$  to  $1$  rd  $s^{-1}$  or  $0.96$  to  $9.6$  rpm.

#### For the module of filtration:

$F_{WS} = 18$ ;

$\Delta P_F = 13332$  to  $66661$  Pa;

$R_{MF} = 1.00 \cdot 10^7$  to  $1.00 \cdot 10^9$   $m^{-1}$ ;

$S_{SP2} = 0.01$  to  $0.04$  kg of dry solids / kg of suspension or  $1$  to  $4\%$ ;

$\alpha = 1.16 \cdot 10^9$  to  $4.02 \cdot 10^9$   $m$   $kg^{-1}$ ;

$\mu_5 = 0.5 \cdot 10^{-3}$   $kg$   $(m$   $s)^{-1}$ ;

$\rho_5 = 10^3$   $kg$   $m^{-3}$ .

#### For the module of washing:

$\rho_8 = 10^3$   $kg$   $m^{-3}$ .

#### For the characteristics of the filter:

$A_F = 10$   $m^2$ ;

$\theta F = 2.5$  rd;

$\theta L = 0.9$  rd.

#### Parameter of the washing mechanism $x_f$ :

$x_f = 0$  to  $1$ .

### RESULTS FOR THE CALCULATION PROCEDURE

From the model that was prepared in the present work and the initial values, a parametric study was performed to demonstrate the influence of some parameters on the efficiency and on the production of the brownstock washing filter.

Figure 2 presents the displacement ratio ( $RD$ ), calculated for different dilution factors ( $FD$ ), as a function of the washing mechanism parameter ( $x_f$ ). It is confirmed that in a situation of plug flow,  $x_f = 1$  the displacement relationship  $RD = 1$ . It can be noticed that there is a direct correlation between  $x_f$  and  $RD$ . In the situation of washing by displacement, for an ideal situation ( $x_f = 1$ ), it is

considered that the volume of water that is employed is equal to the volume of solution that is present in the cake. However, even for the most efficient processes, the volumes of washing liquid that are employed are significantly greater than those in the ideal situation. Such performance can be explained by the flow through the fibrous bed and by the mechanism of diffusion and the mixing of the solute through the fibers, as well as by the processes of adsorption/desorption of the solute in the fibers.

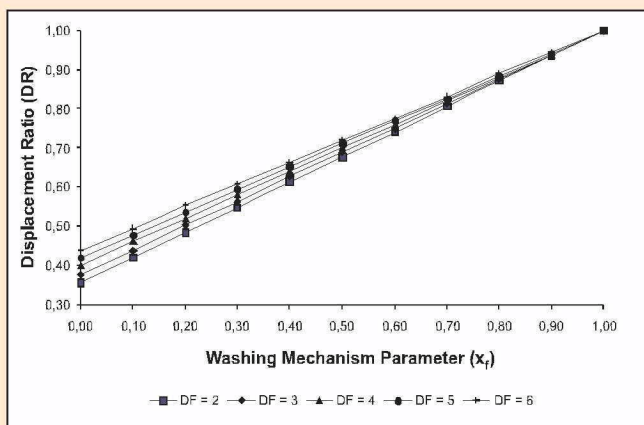
In Figure 3 the same type of result is presented as the loss during the process of washing ( $I-Y$ ). Another classic way of representing the efficiency (or yield) for the process of washing is in the way of the Nordén number ( $E$ ). Figure 4 gives the behavior of  $E$  as a function of the parameters  $FD$  and  $x_f$ .

The results that are illustrated in Figures 2 through 4 allow for an appropriate and effective way of interpreting the macroscopic parameters  $RD$ ,  $FD$  and  $E$ , which are common in the washing area of cellulose, with the parameter  $x_f$  that represents the washing mechanism.

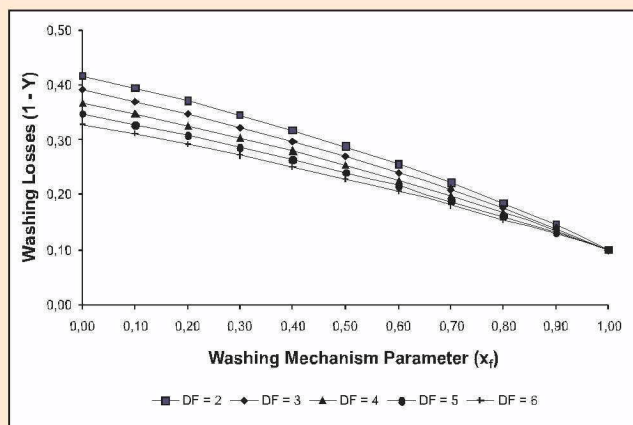
Figures 2 through 4 confirm that the greater the displacement ( $x_f \rightarrow 1$ ) the more efficient will the washing operation be. The results also show that the greater the dilution factor ( $FD$ ), that is, the greater the amount of liquor per pulp that is used, the more efficient will be the washing. The decision regarding the optimal dilution factor will depend on the individual characteristics of each process.

The evaluation of the washing efficiency of a rotary drum vacuum filter, while in operation, presumes, in a previous stage, the full knowledge of the flowrates and concentrations of the streams that refer to the different stages of the process.

Figure 5 illustrates the variation of the fraction of soluble solids ( $X_{S1}$ ) in each unit operation carried on the



**Figure 2.** Displacement ratio ( $RD$ ) as a function of the washing mechanism parameter ( $x_f$ ) for different dilution factors ( $FD$ ). A direct correlation between  $x_f$  and  $RD$  is seen



**Figure 3.** Losses during washing ( $1-Y$ ) as a function of the washing mechanism parameter ( $x_f$ ) for different dilution factors ( $FD$ )

filter for a fixed value of  $x_f = 0.7$ . It is noticed that a large amount of the soluble solids is removed during the operations of dilution and, mainly, washing, so that a good stirring of the vat of the filter is to be guaranteed in order to get a good diffusion of the soluble solids that are present in the cellulosic fibers, as well as an efficient removal of the black liquor during the operation of washing.

Figures 6 through 9 show the influence of the velocity of the filter ( $\omega$ ), of the consistency of the vat of the filter ( $S_{SP2}$ ), and of the resistance of the filtering medium ( $R_{MF}$ ) and the head loss during the process of filtration ( $\Delta P_F$ ) on the specific

productivity of the filter. In all cases were maintained the input values for the fractions of soluble solids, for the properties of the liquor and for the geometry of the filter, for a value of  $FD = 4$  and  $x_f = 0.7$ .

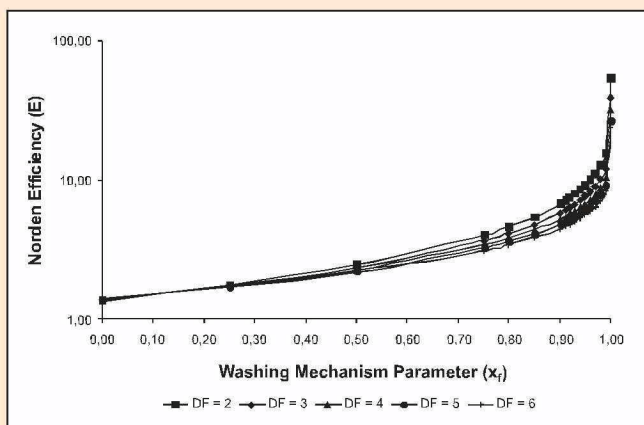
As expected, the greater the angular velocity of the filter ( $\omega$ ), the greater will be its production (Figure 6), but not in the same proportion, due to the decreasing thickness of the cake.

From the results that are shown in Figure 7, the consistency of the vat ( $S_{SP2}$ ) has a large influence on the productivity of the filter. This phenomenon is a consequence of the fact that the greater the consistency of the vat, the lower will be the amount

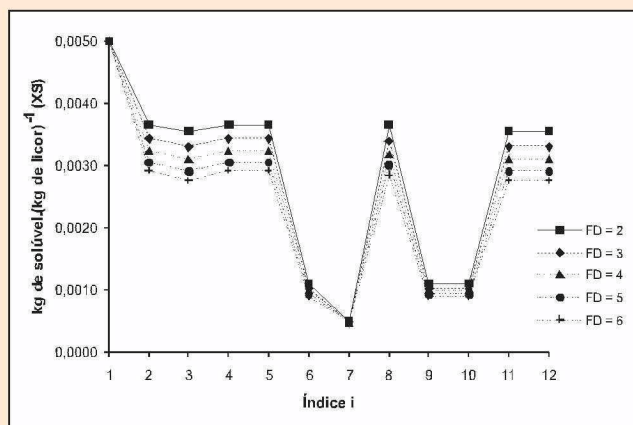
of liquor that will have to be filtrated for a given mass of suspension.

The results that are shown in Figure 8 confirm that the choice of the filtering medium is an important variable for the productivity of the filter. Therefore, in industrial filters, a filtering medium with a head loss that is negligible when compared to the head loss in the cake ( $\Delta P_c$ ) must be the choice. However, it must be paid attention that the filtering medium is capable of guaranteeing the retention of the fibers.

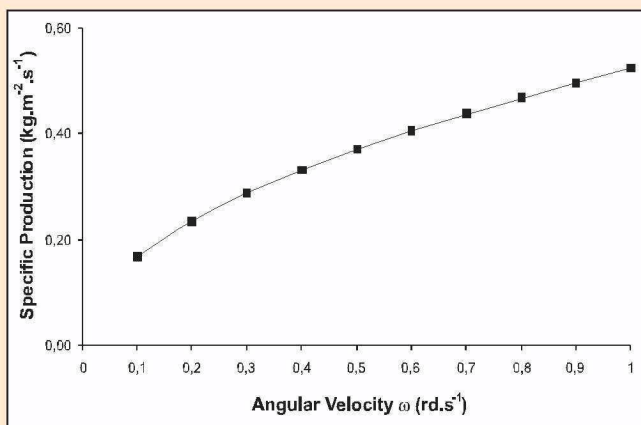
Due to the compressibility of the cellulosic fibers, it is not expected a significant gain in productivity with an increase in the head loss of the filtration ( $\Delta P_F$ ), as it is shown in Figure 9,



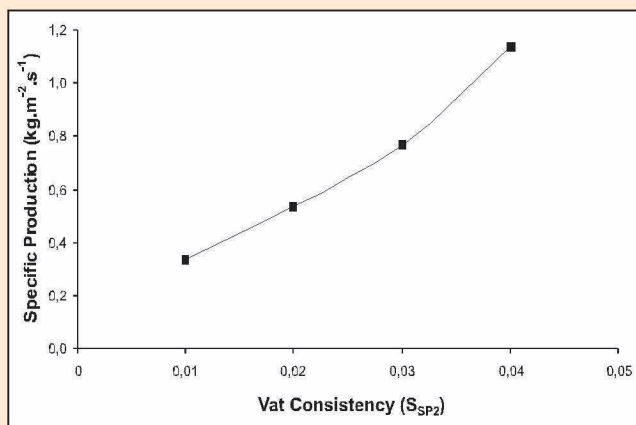
**Figure 4.** Nordén number ( $E$ ) as a function of the washing mechanism parameter ( $x_f$ ) for different dilution factors ( $FD$ ). It can be noticed the correlation between the Nordén number ( $E$ ) and  $x_f$ ,



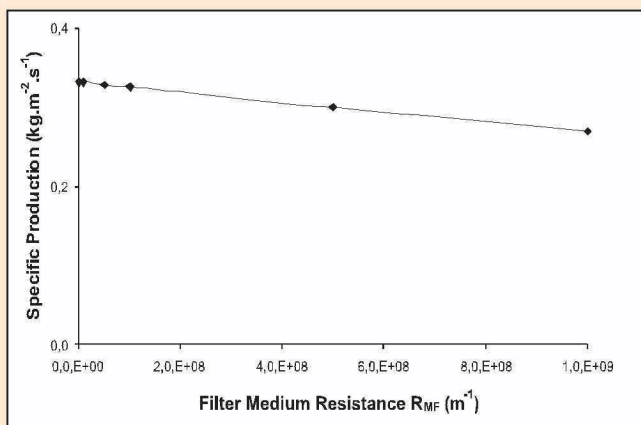
**Figure 5.**  $X_{Si}$  for the different streams in Figure 1 as a function of the dilution factor ( $FD$ )



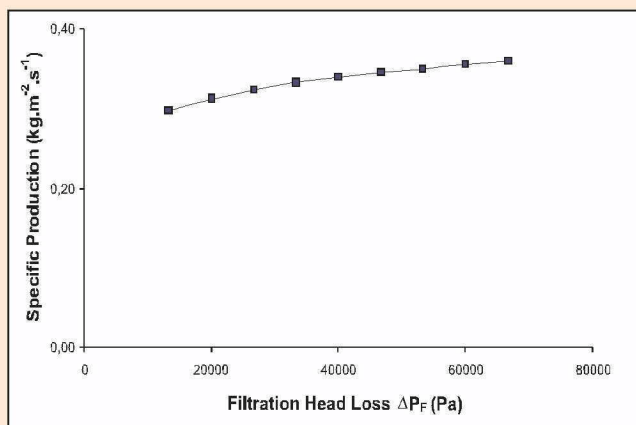
**Figure 6.** Specific production of cellulose as a function of the angular velocity ( $\omega$ )



**Figure 7.** Specific production of cellulose as a function of the consistency of the vat ( $S_{SP2}$ )



**Figure 8.** Specific production of cellulose for different filter medium resistance ( $R_{MF}$ )



**Figure 9.** Specific production of cellulose as a function of the head loss of the filtration ( $\Delta P_F$ ) (Pa)

because an increase in the head loss of the filtrate ( $\Delta P_F$ ) leads to a greater difficulty for the flowing of the fluid through the cake, as a consequence of the reduction in the porosity of the cake, thus characterizing the phenomenon of compressibility in cellulosic cakes.

## CONCLUSIONS

The splitting of the operation of the rotary drum vacuum filter into a modular structure has proven itself important for the understanding of each one of the unit operations that are carried. The combination of the variables in the mass balance of the rotary drum vacuum filter with the fundamental variables of a process of filtration at constant pressure allowed for the development of a model for the process of washing that describes the physical phenomena in the respective models, and they are very

useful in the understanding of the operation of washing.

The definition of the efficiency of the washing through the parameter  $x_f$  provides an alternative to the employment of the parameters  $RD$  and  $FD$ .

The proposed model is an important tool for both the management and the design of vacuum rotary filters for the operation of washing, as it provides a better physical understanding of the unit operations that are taking place at each section of the washing equipment.

## ACKNOWLEDGEMENTS

The authors wish to acknowledge the financial aid received from the CNPq (Process 131359/03-7) and the Indústria de Celulose e Papel RIPASA

S. A. for the opportunity of visiting its facilities, contributing for the current analysis of the washing system.

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## NOMENCLATURA

$A_F$	Filtration area, m <sup>2</sup>
$A_L$	Washing area, m <sup>2</sup>
$E$	Norden number
$f_i$	Fraction of recovered cellulose
$FD$	Dilution factor, kg.kg <sup>-1</sup>
$F_{WS}$	Ratio of mass of wet cake to mass of dry cake, kg.kg <sup>-1</sup>
$GL$	Degree of freedom
$K_C$	Characteristic coefficient of the cake in the filtration section, N.s.m <sup>-8</sup>
$K_{CL}$	Characteristic coefficient of the cake in the washing section, N.s.m <sup>-8</sup>
$K_{MF}$	Characteristic coefficient of the filtering medium in the filtration section, N.s.m <sup>-5</sup>
$K_{MFL}$	Characteristic coefficient of the filtering medium in the washing section, N.s.m <sup>-5</sup>
$M_{Pi}$	Mass flowrate of the cellulosic pulp, kg.s <sup>-1</sup>
$M_{Wi}$	Mass flowrate of water, kg.s <sup>-1</sup>
$RD$	Displacement ratio
$R_{MF}$	Resistance of the filtering medium, m <sup>-1</sup>
$S_{SPi}$	Consistency of the brownstock pulp suspension, kg.kg <sup>-1</sup>
$t_F$	Filtration time, s
$t_L$	Washing time, s
$V_F$	Volume of filtrate, m <sup>3</sup>
$V_L$	Volume of washing liquid during interval of time $t_L$ , m <sup>3</sup>

$W_{Pi}$	Ratio between the mass of water and the mass of cellulose in the suspension, kg.kg <sup>-1</sup>
$x_f$	Washing mechanism parameter
$X_{Si}$	Ratio between the mass of solids and the mass of water in the suspension, kg.kg <sup>-1</sup>
$Y$	Washing efficiency
$Y_L$	Local washing efficiency
$Y_{Li}$	Lower washing efficiency
$Y_{Ls}$	Upper washing efficiency

## GREEK LETTERS

$\alpha$	Specific average resistance of the cake, m.kg <sup>-1</sup>
$\Delta PC$	Head loss in the cake, Pa
$\Delta PF$	Head loss in the filtration section, Pa
$\Delta PL$	Head loss in the washing section, Pa
$\theta_F$	Angle of the filtration section, rd
$\theta_L$	Angle of the washing section, rd
$\lambda$	Number of equations
$\mu_5$	Dynamic viscosity of the filtrate in exit stream n° 5, kg.(m.s) <sup>-1</sup>
$\rho_5$	Density of the filtrate in exit stream n° 5, kg.m <sup>-3</sup>
$\rho_8$	Density of the filtrate in exit stream n° 8, kg.m <sup>-3</sup>
$\sigma$	Number of variables
$\omega$	Angular velocity of the filter cylinder, rd.s <sup>-1</sup>