Statistical process control in paper machine

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ABSTRACT

This paper presents a special statistical process control (SPC) chart suitable for continuous processes like the paper machine in the PPI. Certain aspects that are unique to this kind of process make conventional SPC charts inadequate and misleading, causing false alarms and search of inexistent problems.

INTRODUCTION

Several attempts to introduce Statistical Process Control (SPC) in the paper industry have been unsuccessful. Various are the reasons alleged for this fact, but the main one is the non-acquaintance with certain control charts, referred to as special, destined to control continuous processes, as it is the case of the paper machine.

This paper tries to introduce the control chart called 3-D, as it monitors simultaneously three characteristics of the process. In the end, a comparative example of this one and a traditional chart is presented.

CONVENTIONAL CONTROL CHARTS

The control charts, developed by SHEWHART (1931), admit that a given observation of a quality characteristic X (x_t) obtained from a statistically stable process (with constant average μ and standard deviation σ) can be adequately represented through the mathematical model:

$$x_t = \mu + \varepsilon_t \qquad (1)$$

where εt is the assumed sample related error, **normally and independently** distributed, with average 0 and constant standard deviation.

SHEWHART adopted the criterion that given any statistics (W) calculated based on these observations, independently of its sample related probability distribution, will have as control limits:

$$\begin{aligned} LSC_W &= \mu(W) + 3.\sigma(W) \\ LMW &= \mu(W) \\ LIC_W &= \mu(W) - 3.\sigma(W) \end{aligned} \qquad (2)$$

Due to the fact that the sample related means tend to have a normal distribution, by virtue of the Central Limit Theorem, as well as because the variance of the means is lower than the process variance (or individual values), it is usual to adopt mean (x-bar) and range (R) type charts. The latter are those most frequently used in every kind of industry and the paper making one is no exception.

The control limits for the mean chart are as follows:

$$\begin{aligned} LSC_{\bar{x}} &= \bar{\bar{x}} + A_2.\bar{R} \\ LM_{\bar{x}} &= \bar{\bar{x}} \\ LIC_{\bar{x}} &= \bar{\bar{x}} - A_2.\bar{R} \end{aligned} \qquad (3)$$

where x-two bars is the grand mean of the k samples obtained, A_2 being defined as:

$$A_2 = \frac{3}{d_2.\sqrt{n}} \qquad (4)$$

where d_2 is a correction factor of the bias introduced by the replacement of σ with R-bar in the formula (see Annex A), and R-bar is the mean range defined as:

$$\bar{R} = \frac{\sum R_i}{k} \qquad (5)$$

This chart (x-bar) is used together with the range chart (R), which has as control limits:

$$\begin{aligned} LSC_R &= D_4.\bar{R} \\ LM_R &= \bar{R} \\ LIC_R &= D_3.\bar{R} \end{aligned} \qquad (6)$$

where D_3 and D_4 are also correction factors, a function of the size of the sample (n), supplied in Annex A.

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Further details about these correction factors may be obtained at MONTGOMERY (1996).

Some important comments follow:

a) The control charts are robust as to deviations from normality in the data, as it was demonstrated by BURR (1967) and SCHILLING; NELSON (1976), i.e. even when a process generates data with a distribution that cannot be admitted to be normal, even so the control charts will work satisfactorily.

b) When there is no statistical independence between the data (presence of serial correlation or autocorrelation), the model proposed by Shewhart is unsuitable and may lead to mistakes in the interpretation of the statistical stability of the process. In other words, there is an excessive generation of false alarms, i.e. several points will fall outside the control limits, indicating the presence of a special cause of variation, while in fact this one does not exist.

c) The control chart, which monitors the process centering, makes use of the variation inside the sample to establish the distance of its control limits with regard to the grand mean. Intrinsically it is admitted that the variation represented by R-bar in the formula (4) is suitable to define the amount of variation permitted for the sample related means. It is noted that the distance from the control limits to the mean line in the x-bar chart is a function of R-bar, as well as of factor A_2, which on the other hand depends on the size of sample n.

A CURIOUS CASE STUDY

On a paper machine, in the end of each roll (Jumbo roll), five specimens are taken in machine cross direction. These ones are sent to the laboratory, which determines their basis weight. The results of 25 consecutive rolls are shown in Table 1.

Considering the way the data are presented and taking into account the habitual use of charts for the mean and range (x-bar and R), the natural tendency would be to call every roll a sample and consequently one would go over to calculating the mean (x-bar) and the range (R) per paper roll.

As grand mean (x-two bars) and mean range (R-bar), the following values are obtained:

$$\bar{\bar{x}} = \frac{\sum_{i=1}^{25} \bar{x}_i}{25} = 74{,}9$$

$$\bar{R} = \frac{\sum_{i=1}^{25} R_i}{25} = 3{,}09$$

Applying these results to the formulas (3) and (5) and remembering that in this case n = 5, the control charts of Figure 1, shown in the following, are obtained.

The control chart of the mean indicates the presence of a special cause of variation: stratification. In other words, the chart points to an apparently curious problem, which is the lack of variation in the process. When stratification appears, its cause is usually either in the way the samples have been collected or else, how they have been applied to the control limit calculation. In the particular case, the charts have been set up without analyzing which kind of variation is being pointed to on each of them.

The range (R) chart always presents a variation called within sample, i.e. in the present situation the basis weight variation in machine cross direction, since all five specimens are thus obtained. On

Table 1 - Paper roll data

Roll	Values					Mean	Range
1	73,9	74,6	76,4	75,8	73,3	74,8	3,1
2	73,0	75,0	76,8	75,3	73,0	74,6	3,8
3	74,1	74,5	75,4	74,5	73,3	74,4	2,1
4	74,6	74,4	76,0	76,0	73,9	75,0	2,1
5	74,1	75,6	75,6	75,8	73,5	74,9	2,3
6	73,7	75,1	77,0	75,3	73,4	74,9	3,6
7	74,1	74,9	76,1	75,6	73,8	74,9	2,3
8	74,0	74,9	76,2	76,5	73,2	75,0	3,3
9	74,2	75,4	77,4	75,3	73,2	75,1	4,2
10	74,1	74,7	75,7	75,7	73,5	74,7	2,2
11	74,2	75,9	75,8	75,4	73,2	74,9	2,7
12	73,7	75,6	76,5	76,1	74,2	75,2	2,8
13	73,2	75,6	75,7	76,7	73,0	74,8	3,7
14	73,3	76,1	75,9	76,0	73,3	74,9	2,8
15	74,3	75,0	77,0	76,2	73,8	75,3	3,2
16	73,8	74,8	76,3	75,6	73,7	74,8	2,6
17	74,3	74,7	75,4	75,7	72,9	74,6	2,8
18	74,2	74,9	76,5	75,7	72,9	74,8	3,6
19	72,9	74,6	75,6	76,6	73,3	74,6	3,7
20	74,8	75,1	75,7	75,9	73,0	74,9	2,9
21	74,1	74,6	76,3	76,3	73,5	75,0	2,8
22	73,7	74,7	75,3	76,6	73,2	74,7	3,4
23	74,2	75,7	75,2	76,5	73,5	75,0	3,0
24	73,5	75,7	76,3	76,5	72,7	74,9	3,8
25	73,3	74,3	77,1	75,5	72,9	74,6	4,2

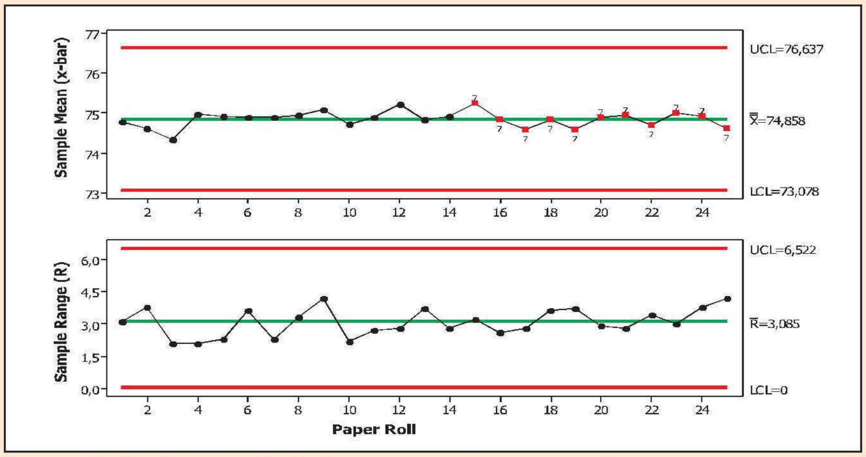

Figure 1 - Control charts for basis weight means and ranges

the other hand, the mean chart (x-bar) shows another type of variation, called between samples, i.e. the basis weight variation in machine direction.

It is known by the practice in controlling this process that the machine cross direction variations are of a totally distinct nature from that of the machine direction variations. Consequently, it can be said that using the machine cross direction variation (represented here by R-bar) to establish how much the process can vary in machine direction (x-bar) is a totally mistaken practice.

3-D CONTROL CHART

On a conventional chart for variables, the R-bar value determines the distance at which the control limits are from the mean line on the x-bar chart. In other words, the variation within sample determines the difference that can exist in the variation between samples, before the latter is considered as statistically significant.

However, there are situations in which the variation inside the sample does not serve as a good basis for establishing the control limits of x-bar. Cases where this occurs are:

• in batch manufacturing of lots, where the differences between lots are pronounced due to the inherent raw material variation and there is no possibility of reducing it;

• in continuous product manufacturing (paper machine, for instance), where the machine cross direction variation is no suitable basis to establish the machine direction variation range due to their completely opposite natures.

The 3-D (three-dimensional) control chart is, as a matter of fact, a combination of the mean and range charts (x-bar and R) with the charts for individual values and moving range (x-MR), according to RAMOS (2000), so that they make possible to simultaneously control more than two types of variation.

The R chart will monitor the variation within sample. Consequently:

$$LSC_R = D_4 . \overline{R}$$
$$LM_R = \overline{R} \qquad (7)$$
$$LIC_R = D_3 . \overline{R}$$

The MR chart, for its part, will serve as a basis to establish the distance of the control limits to the mean line on the x-bar chart. Therefore:

Table 2 - Basis weight values of 25 paper rolls

Roll	Values					Mean	Range	Moving Range
1	73,9	74,6	76,4	75,8	73,3	74,8	3,1	-
2	73,0	75,0	76,8	75,3	73,0	74,6	3,8	0,2
3	74,1	74,5	75,4	74,5	73,3	74,4	2,1	0,3
4	74,6	74,4	76,0	76,0	73,9	75,0	2,1	0,6
5	74,1	75,6	75,6	75,8	73,5	74,9	2,3	0,1
6	73,7	75,1	77,0	75,3	73,4	74,9	3,6	0,0
7	74,1	74,9	76,1	75,6	73,8	74,9	2,3	0,0
8	74,0	74,9	76,2	76,5	73,2	75,0	3,3	0,1
9	74,2	75,4	77,4	75,3	73,2	75,1	4,2	0,1
10	74,1	74,7	75,7	75,7	73,5	74,7	2,2	0,4
11	74,2	75,9	75,8	75,4	73,2	74,9	2,7	0,2
12	73,7	75,6	76,5	76,1	74,2	75,2	2,8	0,3
13	73,2	75,6	75,7	76,7	73,0	74,8	3,7	0,4
14	73,3	76,1	75,9	76,0	73,3	74,9	2,8	0,1
15	74,3	75,0	77,0	76,2	73,8	75,3	3,2	0,3
16	73,8	74,8	76,3	75,6	73,7	74,8	2,6	0,4
17	74,3	74,7	75,4	75,7	72,9	74,6	2,8	0,2
18	74,2	74,9	76,5	75,7	72,9	74,8	3,6	0,2
19	72,9	74,6	75,6	76,6	73,3	74,6	3,7	0,2
20	74,8	75,1	75,7	75,9	73,0	74,9	2,9	0,3
21	74,1	74,6	76,3	76,3	73,5	75,0	2,8	0,1
22	73,7	74,7	75,3	76,6	73,2	74,7	3,4	0,3
23	74,2	75,7	75,2	76,5	73,5	75,0	3,0	0,3
24	73,5	75,7	76,3	76,5	72,7	74,9	3,8	0,1
25	73,3	74,3	77,1	75,5	72,9	74,6	4,2	0,3

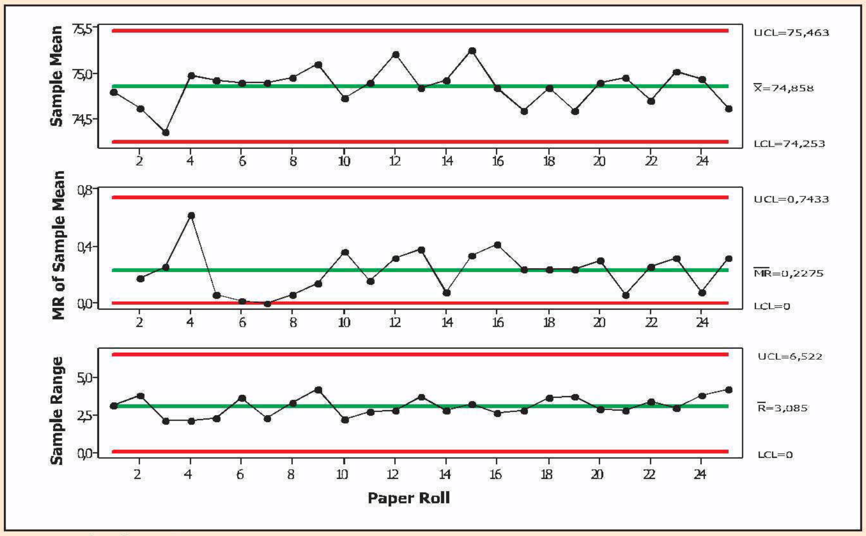

Figure 2 - 3-D Graph

$$LSC_{Rm} = D_4.\overline{Rm}$$
$$LM_{Rm} = \overline{Rm} \qquad (8)$$
$$LIC_{Rm} = D_3.\overline{Rm}$$

Finally, the x-bar chart will be calculated by means of the formulas:

$$LSC_{\bar{x}} = \bar{\bar{x}} + E_2.\overline{Rm}$$
$$LM_{\bar{x}} = \bar{\bar{x}} \qquad (9)$$
$$LIC_{\bar{x}} = \bar{\bar{x}} - E_2.\overline{Rm}$$

THE CURIOUS CASE REVISITED

Since the control of variation in both machine and machine cross directions are important, the 3-D graph is a sensible option. The same data of Table 1 are re-presented in the following (Table 2). However, a new column was added, that of the moving range of values taken in pairs (the modulus of the highest minus the lowest value).

Following is obtained with the results of this last table:

$$\overline{Rm} = \frac{\sum_{i=2}^{25} Rm_i}{24} = 0,23$$

The remaining grand mean and mean range values remain unchanged. The control limits for the MR chart are:

$$LSC_{\bar{x}} = \bar{\bar{x}} + E_2.\overline{Rm} = 74,9 + 2,660 * 0,23 = 75,5$$
$$LM_{\bar{x}} = \bar{\bar{x}} = 74,9$$
$$LIC_{\bar{x}} = \bar{\bar{x}} - E_2.\overline{Rm} = 74,9 - 2,660 * 0,23 = 74,3$$

The 3-D chart is supplied in Figure 2. By analyzing the control graphs, it can be remarked that the process is stable (there is no presence of special variation causes having an influence).

CONCLUSIONS

Although the books about traditional quality tools usually do not present this technique, it is extremely important in the pulp and paper industry (PPI), as on the paper machine. Even in leading statistical softwares this control chart is not easily found, or else it is presented by another name.

An ill-selected tool may cause several problems in the quality control and improvement. Thus, using conventional mean and range (x-bar and R) control charts will lead to wrong conclusions. The 3-D control chart is a suitable solution.

LITERATURE

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Annex A - Factors for control limit calculation

n	A_2	A_3	E_2	B_3	B_4
2	1,880	2,695	2,660	-	3,267
3	1,023	1,954	1,772	-	2,568
4	0,729	1,628	1,457	-	2,266
5	0,577	1,427	1,290	-	2,089
6	0,483	1,287	1,184	0,030	1,970
7	0,419	1,182	1,109	0,118	1,882
8	0,373	1,099	1,054	0,185	1,815
9	0,337	1,032	1,010	0,239	1,761
10	0,308	0,975	0,975	0,284	1,716

n	D_3	D_4	D	c_4	d_2
2	-	3,267	0,709	0,798	1,128
3	-	2,574	0,524	0,886	1,693
4	-	2,282	0,446	0,921	2,059
5	-	2,114	0,403	0,940	2,326
6	-	2,004	0,375	0,952	2,534
7	0,076	1,924	0,353	0,959	2,704
8	0,136	1,864	0,338	0,965	2,847
9	0,184	1,816	0,325	0,969	2,970
10	0,223	1,777	0,314	0,973	3,078

Fonte: MONTGOMERY, D.C. Introduction to statistical quality control. 3 ed. New York, John Wiley, 1996.