

# REVIEW OF PHYSICAL PRINCIPLES IN LOW CONSISTENCY REFINING

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## ABSTRACT

Considering that low consistency refining is fundamental to papermakers and to researchers as well, the present paper intends to review its physical principles starting from the first attempts that culminated in the C-factor theory, which was closely examined. Forces in refining are the prime variable and their definitions and effects are described as well. The low consistency refining is essentially seen by authors as fatigue phenomenon due its cyclic nature, and some results and concepts from fatigue theory are used and applied. The damage variable was introduced in order to have an accessible tool to describe refining. Common effects of refining, as for instance internal and external delamination, are explained as consequences of pulp fiber fatigue.

## INTRODUCTION

Refining is a fundamental operation in the manufacture of paper considering its importance on the final paper properties. It is the only operation that modifies pulp fiber morphology, shaping the paper with the desired optical, physical and printing properties. At the present time, with paper machines running at 2000 m/min, it is of greatest importance that refining should be a fully known and controlled process, so as not to adversely affect machine productivity or exceed planned output costs. Refining can be described as an operation which, through transmission of mechanical energy from the refining plant, irreversibly changes the structural morphology of pulp fibers. This is done by passing an aqueous suspension of fibers through the gap between two slotted surfaces, one of which is stationary (stator) and the other moving at high velocity (rotor), with a minimum distance between them, so that energy of friction is transmitted providing the necessary rubbing to refine the fiber. Despite various experimental publications, very few have attempted to understand the physical nature of refining.

Since the 1960s Derek H. Page, in a series of outstanding studies, examined, among other issues, the effect of delamination and collapse of fibers under refining conditions outlining an analytical description of the problem in which initial results came from the

semiempirical models of the 1940s. In 1977, Leider and Nissan proposed an analytical model describing refining as a combination of the number of impacts received by a fiber and their intensity. All these investigations were culminated in 1990 with the publication of the C-factor theory of Richard J. Kerekes. More recent experimental work of Hamad (1997, 1998) has related refining directly to fiber fatigue. Kerekes (2011) reviewed several publications on energy and forces in refining.

The aim of the present study is reviewing the physical basis of the phenomenon of refining, being emphasized the fatigue of fibers, according the authors' point of view and following it also about fundamental forces on refining. The real impact of the forces has several variables such as stochastic distribution of fibers, quality of flocs and fibrillations formation, flash vaporization of water, fiber suspension flow, sharp temperature gradient of the industrial refiners, fiber morphological characteristics, pH swelling of fibers, and so on. Nowadays all of these aspects are subjects of macroscale laboratory experiments, but the multiscale translation is still beyond the state of the art of the mathematical modeling based on physical principles.

## MATHEMATICAL MODELS OF REFINING

The first attempt to describe refining mathematically dates back the end of the 19<sup>th</sup> century by Jagenberg, who pioneered it by introducing concepts like "edge length per second" and "beating area". We call macromodels as being models who relate energy of refining, number of impacts, quantity and intensity of refining with fiber or paper end properties. Macromodels also describe the hydrodynamics of fiber suspensions in the interior of the refining disc.

If we want to look at refining as a fatigue phenomenon, one important starting point is reckoning the average number of impacts  $N$  received by fibers or fiber flocs during refining. For a disk refiner, and following Leider and Nissan (1977) footsteps, firstly thing is to postulate that  $N$  is described by the equation:

$$N = f\left(n_r, n_s, w, \tau^f, \frac{A_f}{A_g}\right) \quad (1)$$

where  $N$  is the number of impacts,  $n_r$  the number of bars on the rotor arm,  $n_s$  the number of bars on the static disc,  $w$  the rotational velocity

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of rotor disc,  $\tau^f$  the time a fiber remains in the refiner,  $A_f$  the mean fiber straight section longitudinal area, and  $A_g$  the straight-section groove area. The term  $(A_f/A_g)$  estimates, in principle, probability of a fiber coming into contact with refiner bars during refining. This probability explains the unpredictable character of refining in which some fibers may come all the way through the equipment without being refined.

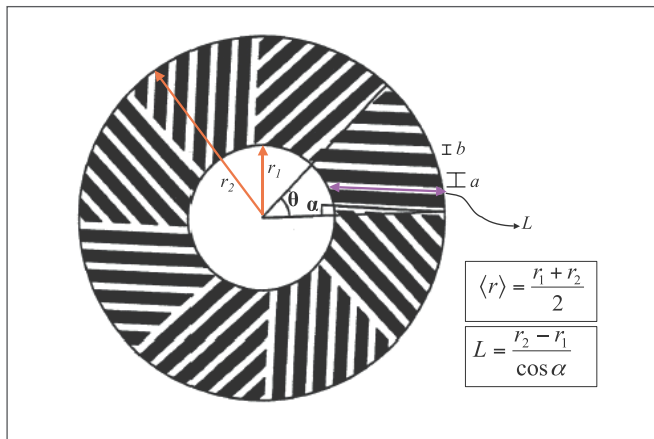
The simplest supposition is to express the equation with directly proportional variables:

$$N = n_r n_s w \tau^f \left( \frac{A_f}{A_g} \right) \quad (2)$$

The number of static and rotor bars can be calculated:

$$n \text{ or } n_s = \frac{2\pi \langle r \rangle}{a+b} = \frac{\pi \langle D \rangle}{a+b} \quad (3)$$

where  $\langle r \rangle$  and  $\langle D \rangle$  are average and internal radius of the refining area – and average disc diameter respectively,  $a$  is rotor or static bar width, and  $b$  is groove width, as shown in **Figure 1**.



**Figure 1.** Geometric refiner disc parameters

Estimated time in the refiner is calculated by dividing groove length  $L$  (figure 1) by mean fiber velocity in the groove:

$$\tau^f = \frac{L}{\langle u_c \rangle} = \frac{L A_d}{q} \quad (4)$$

where  $q$  is volumetric pulp draining and  $A_d$  is available flow area, equivalent to:

$$A_d = (2\pi \langle r \rangle) (2c + h) A_{util} \quad (5)$$

Here,  $c$  is groove depth,  $h$  is distance between discs and  $A_{util}$  is effective flow area; in other words, area taken up by grooves plus the empty space between discs.

Considering that  $a$  equals  $b$ , it may be concluded that  $A_{util}$  equals  $1/2$ , so:

$$A_d = \pi \langle r \rangle (2c + h) \quad (6)$$

The time during which a fiber remains in a groove may be expressed as:

$$\tau^f = \left( \frac{r_2 - r_1}{\cos \alpha} \right) \frac{\pi \langle r \rangle (2c + h)}{q} \quad (7)$$

where the angle  $\alpha$  is the bar angle (Figure 1). The coefficient  $(A_f/A_g)$  can be calculated, considering first that  $A_f \equiv D_f L_f$ , where  $D_f$  is fiber diameter and  $L_f$  is its length. As  $A_g = bc$ , it follows that:

$$\frac{A_f}{A_g} = \frac{D_f L_f}{bc} \quad (8)$$

Substituting equations (3), (7) and (8) for (2), we arrive at an expression for mean number of impacts:

$$\langle N \rangle = 4\pi^3 \langle r \rangle^3 \left[ \frac{\left( 2 + \frac{h}{c} \right)}{b(a+b)^2} \right] \left( \frac{r_2 - r_1}{\cos \alpha} \right) \frac{w}{q} D_f L_f \quad (9)$$

In this way, the net energy transmitted to fibers can be calculated. Other important macromodels are as follows:

- A theory of specific edge load (SEL) developed by Brecht and Siewert in 1966, a theory based on the hypothesis that the nature of the refining action of any low consistency refiner is described by specific edge load, which is defined as the net energy input divided by length of cutting bars per unit time; in other words: the SEL is a measure of the energy expended per unit length of bar crossings.

- The theory of specific surface charge (SSC), which was developed in its final form by Lumiainen in 1990, can be seen as an improvement of SEL. The theory emphasizes that bar width is a fundamental factor in refining action and that length of bar impact is a parameter that must be included in qualitative refining calculations. The central concept of this theory is that energy is transferred to the fiber mainly during the edge/edge and edge/surface steps.

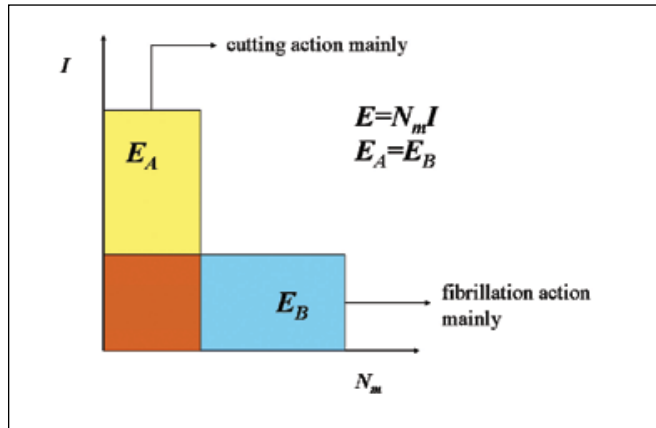
### C-FACTOR

Developed in 1990 by Kerekes, the C-factor macromodel is the most rigorous refining theory currently in use. The starting point is similar to other theories: net refining energy per unit mass may be calculated from the number of impacts multiplied by energy:

$$E_{net} = N_m I \quad (10)$$

where  $N_m$  is number of impacts per mass and  $I$  is energy transferred per impact. We observe, in principle, that net energy specified by  $E_{net}$  does not give all of the refining information, and so we must consider two different operations acting in refining: the first being a large number of impacts of low intensity, and the second the reverse situation, a small number of high intensity impacts. The first gives

rise to *fibrillation* of fibers, and the second the *cutting* of fibers. Fibrillation and cutting result from very different refining processes – as every papermaker knows - but numerically the specific net energy is the same. This is illustrated in **Figure 2**.



**Figure 2.** Cutting and fibrillating actions of refiner

The two actions of refining are equivalent only when parameters  $N$  and  $I$  are equal, and it is fundamental in C-factor theory to be able to calculate these two parameters individually to allow, in each case, analysis of its preferential effect in refining. The theory's main idea is to introduce a third variable, the C-factor, into equation 10:

$$E_{net} = \left( \frac{C}{q_m} \right) \left( \frac{P_{net}}{C} \right) \quad (11)$$

with

$$N_m = \left( \frac{C}{q_m} \right) \quad \text{and} \quad I = \left( \frac{P_{net}}{C} \right) \quad (12a), (12b)$$

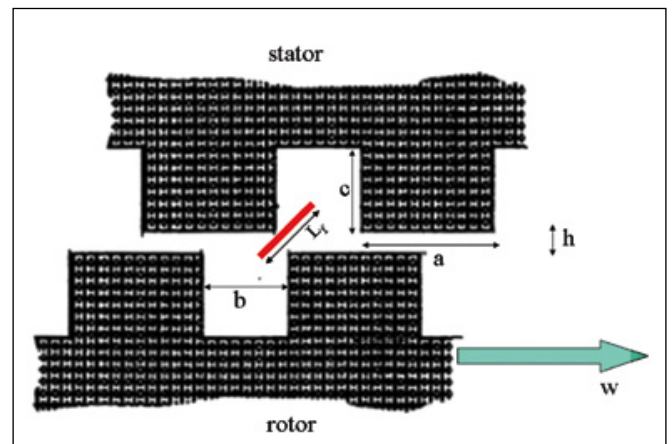
where  $q_m$  is the refiner fiber-mass draining capacity and  $P_{net}$  is the net transferred potential. The C-factor is, in summary, a measure of refiner's capacity to impose impacts onto fibers. The starting point is the calculation of the number of impacts per unit dry mass:

$$N_m = \frac{N}{M} \quad (13)$$

where  $M$  is mass of fiber. The number of impacts  $N$  received by the fiber is proportional to the number of contacts between rotor and stator edges (number of crossings) and the probability that fiber impact occurs with each crossing.

From this idea, the number of impacts per unit time  $dN/dt$  may be expressed in proportion to the total number of crossings with respect to any particular radial position  $r$ . The key is to correctly calculate the proportionality. **Figure 3** shows the parameters involved in the refining process.

Knowing that proportions are constant, the maximum number of impacts is produced when all crossings impose impact on the fiber, in



**Figure 3.** Dimensions in low consistency refining

other words, maximum number of impacts along the fiber's length. This may be calculated by the fiber length and total circle length in radial position  $r$  ratio, literally  $L_f / 2\pi r$ . The maximum value will therefore be:

$$\frac{dN_{max}}{dt} = \frac{L_f n_r n_s w}{2\pi r} \quad (14)$$

Physically, this condition is met when the fiber is oriented tangentially to the bars (condition of maximum impact). All fibers do not, however, undergo this treatment; they may be in grooves or in positions of partial impact. The probability for a fiber to be in contact with edges depends on relative size of fiber length, groove depth and distance between discs. So, the probability can be described by the fraction  $L_f / (L_f + c + h)$ , which reaches a maximum value of 1 when a fiber is very long, and it becomes small when, for example, groove depth is great. The mathematical expression for number of impacts per unit time may then be calculated as:

$$\frac{dN}{dt} = \left( \frac{L_f}{L_f + c + h} \right) \left( \frac{L_f n_r n_s w}{2\pi r} \right) \quad (15)$$

Next, two cases are possible: firstly, Equation 15 reduces to 14 when  $L_f \gg (c+h)$ . Assuming it and that  $L_f \cong (a+b)$  as well, noticing that  $n_r(a+b) = 2\pi r$  and replacing it in equation 15 results:

$$\frac{dN}{dt} = n_s w \quad (16)$$

Let us now consider another condition when  $L_f \ll (c+h)$ , a small fiber in relation to groove depth and interdisc distance. Substituting in Equation 15 gives:

$$\frac{dN}{dt} = \frac{L_f^2 n_r n_s w}{2\pi r (c+h)} \quad (17)$$

Interesting to point out that when fibers are small they tend to agglomerate into flocs, which in turn have dimensions of  $L_f^2$ . It can therefore be induced that, when fibers are small or when grooves

are very deep, the dimensions of flocs formed play an important role in calculating the number of impacts calculus. Finally, it may be considered an intermediate case where  $L_f = a = b = c$ , and  $L_f \gg h$ . In this case,  $2L_f n_r = 2\pi r$ . Substitution into equation 15 now gives:

$$\frac{dN}{dt} = \frac{n_s w}{4} \quad (18)$$

which is a quarter of Equation 16. In this case the number of impacts is fewer because some of the fibers are in grooves, and not subject to impacts.

With  $dN/dt$  defined, we can calculate number of impacts per fiber. To do this, let us consider a  $dr$  increase in the direction of the radius, within the area of refining. Analogous to Equation 4 we have:

$$dt = dr \frac{A_d}{q} \quad (19)$$

where  $q$  is the volumetric pulp flow, and  $A_d$  is the available area for flow within the circumference  $2\pi r$ . From Figure 3, expressing indices  $r$  and  $s$  as rotor and stator respectively, it can be noticed that:

$$A_d = 2\pi r h + n_r c_r b_r + n_s c_s b_s \quad (20)$$

Introducing the variable  $n_{r,s} = \frac{n_{r,s}}{2\pi r}$  for rotor density at unit arc length, the above equation may be rewritten as:

$$A_d = 2\pi r (h + n_r^* c_r b_r + n_s^* c_s b_s) \quad (21)$$

Substituting (21) for (19), and this for (15), we have:

$$dN = dr 4\pi r^2 L_f^2 n_r^* n_s^* w \frac{(h + n_r^* c_r b_r + n_s^* c_s b_s)}{q(L_f + c + h)} \quad (22)$$

In this equation we find that the  $c$  term has no index reference; so,  $c$  must be understood as an arithmetic average of  $c_r$  and  $c_s$ . A problem arises from the number of rotor bar crossings; a refiner disc is divided into segments, the bars of which, both rotor and stator, are angled in relation to their radial direction. Because of this, for every crossing of bars at circumference  $2\pi r$  there is an additional number of crossings with radial increment  $dr$ , determined by the average angle of bars in each segment, as can be seen in **Figure 4**.

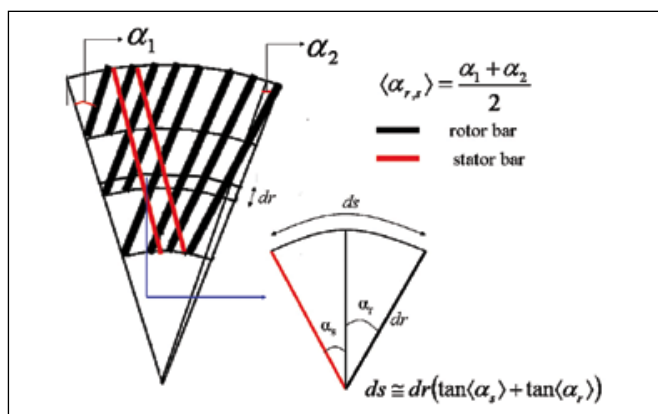


Figure 4. Bar-crossing effect

From Figure 4 can be noticed that:

$$n_{r,s}^* ds = n_{r,s}^* dr (\tan(\alpha_s) + \tan(\alpha_r)) \quad (23)$$

Equation 23 is valid for a single rotor bar; taking into account the total number of crossings within the circumference  $2\pi r$ , the additional number of crossings due to the "non radiality" of the rotor and stator bars becomes:

$$dN_{additional} = n_s^* n_r^* dr (\tan(\alpha_s) + \tan(\alpha_r)) \quad (24)$$

As would be expected, when  $\langle \alpha_r \rangle = \langle \alpha_s \rangle = 0$ , Equation 24 is null. For this reason the term  $n_s^* n_r^*$  in Equation 22 should be substituted for  $n_s^* n_r^* + n_s^* n_r^* (\tan(\alpha_s) + \tan(\alpha_r))$ , which gives the following:

$$dN = dr 4\pi r^2 L_f^2 n_r^* n_s^* w (1 + (\tan(\alpha_s) + \tan(\alpha_r))) \frac{(h + n_r^* c_r b_r + n_s^* c_s b_s)}{q(L_f + c + h)} \quad (25)$$

The above equation may be integrated into the refining area, which means, into  $r_1$  to  $r_2$  as indicated in Figure 1:

$$\int_{r_1}^{r_2} dN = N = 4\pi L_f^2 n_r^* n_s^* w (1 + (\tan(\alpha_s) + \tan(\alpha_r))) \frac{(h + n_r^* c_r b_r + n_s^* c_s b_s)}{q(L_f + c + h)} \frac{(r_2^3 - r_1^3)}{3} \quad (26)$$

The mass  $M$  of fiber can be expressed by  $M = Lf \rho_w$ , where  $\rho_w$  is the linear density of the fiber (coarseness). Mass flow  $q_m$  equals  $q_m = C_F q$ , where  $C_F$  is the consistency and  $q$  the flow rate of the pulp suspension. The C-factor of a disc refiner may therefore be calculated from equations 12a, 13 and 26:

$$C_{disc} = 4\pi^2 L_f C_F n_r^* n_s^* w (1 + (\tan(\alpha_s) + \tan(\alpha_r))) \frac{(h + n_r^* c_r b_r + n_s^* c_s b_s)}{\rho_w (L_f + c + h)} \frac{(r_2^3 - r_1^3)}{3} \quad (27)$$

For the great majority of disc refiners,  $c \gg h$  and the disc design is the same for rotor and stator ( $n_r^* = n_s^*$ ,  $c_r = c_s$ ,  $b_r = b_s$  and  $\alpha_s = \alpha_r$ ), so, Equation 27 may be simplified to:

$$C_{disc} = 8\pi^2 L_f C_F n^3 w c b (1 + 2 \tan(\alpha)) \frac{(r_2^3 - r_1^3)}{3\rho_w (L_f + c)} \quad (28)$$

From this equation it can be seen the dependence of C-factor on some fundamental properties. It is directly proportional to pulp consistency  $C_F$ , the rotation velocity  $w$  of the disc, and the difference  $r_2^3 - r_1^3$ ; is inversely proportional to linear density  $\rho_w$  of the fiber. In the similar way, it is possible to calculate the C-factor for a conical refiner. For more details we refer the reader to Kerekes (1990).

#### FORCES

Although energy being an easier and a more commonly variable to handle in the paper mills, force is the prime variable in low

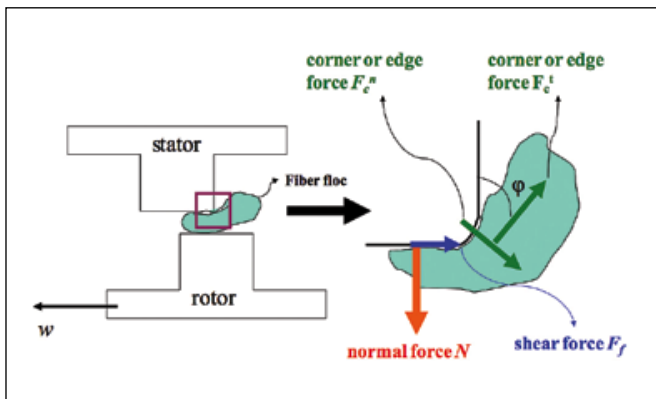


Figure 5. Forces on low consistency refining

consistency refining. It is largely accepted that are three the forces acting in refining: normal force, shear force and corner or edge force, which are shown in **Figure 5**, where fibers are seen like flocs.

Normal force is directly related to the compression of flocs between stator and rotor bars, while shear force is the hydrodynamical friction force among the surface bars and flocs bringing about external delamination (see **Figure 6**) increasing the number of bonding between fibers, thus enhancing paper strength. Corner force is the force produced by the edge bars, being stronger as much sharper is the edge. This force has generally a great intensity and short duration, and in many cases is the cause of internal delamination.

These forces were mathematically defined by Martinez *et al.* (1997) for normal forces, Batchelor (1997) for shear forces and corner forces, and for a general broaden description see Roux (2001) and more recently (2007).

### VISCOELASTIC PROPERTIES OF PULP FIBERS

Fiber walls are composed of layers. Externally, these are the L, S1 and S2 layers. The L layer is normally lost during chemical pulping. The S2 layer is much thicker (5 to 20 times) than the others, being this layer that provides to fiber its mechanical properties. A pulp fiber may be thought as a reinforced compound material;

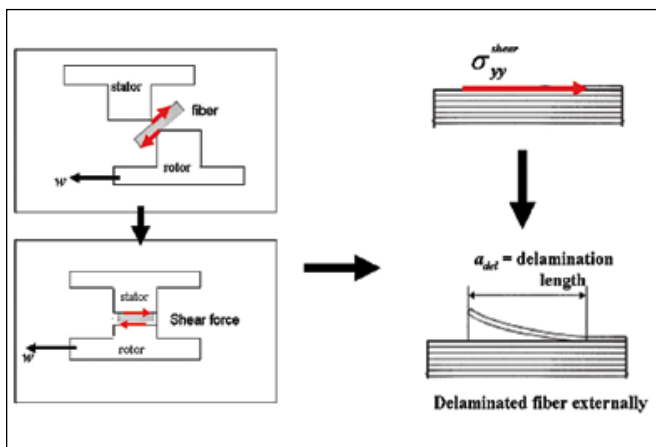


Figure 6. Schema of external delamination fatigue

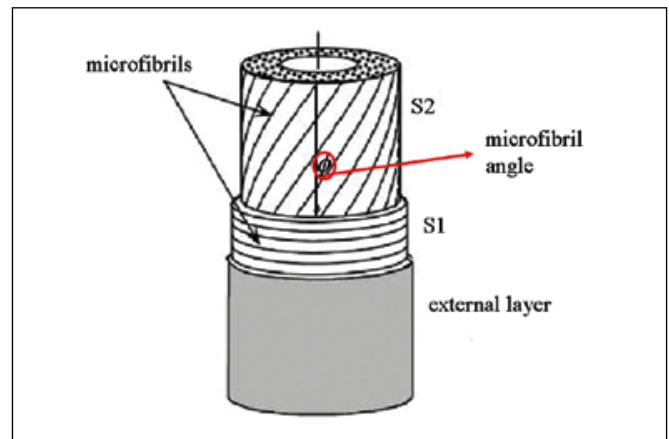


Figure 7. Pulp fiber model showing internal structure and microfibril angle

hollow amorphous matrix (lumen) composed of natural, branched short-chain polymers, mainly hemicellulose and lignin, enclosed in spiral coating of microfibrils, which are crystalline cellulose agglomerates approximately 10 – 30 nm long, each of which contains, in straight section, between 2 and 30,000 molecules of cellulose (Eichhorn *et al.*, 2001). This microfilament coating is at a constant angle  $\varphi$  with respect to the main axis of the fiber and is directly related to its longitudinal and transverse elasticity modulus. Thus, the fiber is a compound material made up of a "hard" phase of high resistant and elastic modulus surrounding a ductile matrix of weak mechanical properties. Layer S1 is also made up of microfibril spirals; in this model the fiber is cylindrical and radially symmetrical as shown in **Figure 7**.

Pulp fibers, like all woods, show viscoelastic properties, that is, combine properties of an elastic solid with those of a viscous fluid. In the study of such materials two functions are very important: the creep  $J(t)$ , which is time dependent change in strain following a step change in stress, and the stress relaxation modulus  $G(t)$ , which is the behaviour of a linear viscoelastic material that, when subjected to a constant strain, will relax under constant strain, so that the stress gradually decreases. Using the Boltzmann superposition principle – see Ward and Sweeney (2004), for instance - these two properties can be mathematically expressed as:

$$\varepsilon(t) = \int_0^t J(t-\tau) \frac{d\sigma(\tau)}{d\tau} d\tau, \sigma(t) = \int_0^t G(t-\tau) \frac{d\varepsilon(\tau)}{d\tau} d\tau \quad (29a), (29b)$$

where  $\varepsilon(t)$  is strain and  $\sigma(t)$  is stress.

As can be seen in the temporal functions described above, when dealing with fatigue of viscoelastic substances load frequency is an important parameter. Although the number of cycles is relevant, in the case of refining the number of impacts imparted by the bars on the fiber or the time at which these cycles are subjected – in other

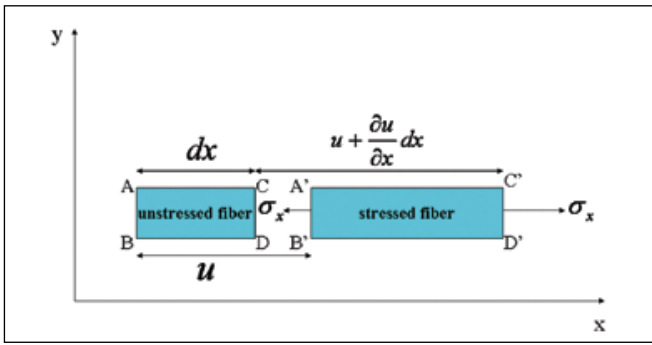


Figure 8. Fiber undergoing axial force

words: frequency – is still more relevant, as Nielsen (2000) explains. From this theoretical model Dunford and Wild (2002) carried out an experiment in which the transverse compression of pulp fibers under cyclic tension was studied.

### ELASTICITY AND FATIGUE THEORY APPLIED TO LOW CONSISTENCY REFINING

A simple result from elasticity theory is the mathematical calculation of impact energy from refiner bars absorbed by one unit mass of fiber through stretching. We will take fibers as being ideally rectangular with resultant tension applied axially, and the effect of inertia will be ignored so that the fiber is in constant equilibrium.

In **Figure 8**,  $u$  is the displacement vector; fiber ABCD is displaced to A'B'C'D'.  $AB \rightarrow A'B'$  with displacement  $u$ ; so,  $CD \rightarrow C'D'$  displacement equals  $u + \frac{\partial u}{\partial x} dx$ . The term  $\frac{\partial u}{\partial x}$  relates to Hooke's law in the following way:

$$\frac{\partial u}{\partial x} = \epsilon(x) = \frac{\sigma_x}{E_{long}} \quad (30)$$

where  $\epsilon$  is strain,  $\sigma$  is tension, and  $E_{long}$  is elasticity or Young's modulus, longitudinal to the fiber.

The work carried out on the fiber, which is equal to the strain energy stored, can be calculated, noting that work carried out in the segment AB is negative, as it is in the opposite direction to the displacement:

$$dU = \int_0^{\sigma_x} \sigma d \left( u + \frac{\partial u}{\partial x} \right) dydz - \int_0^{\sigma_x} \sigma du dydz = \int_0^{\sigma_x} \sigma d \left( \frac{\partial u}{\partial x} \right) dx dy dz \quad (31)$$

Substituting Equation 29 for Equation 30 and integrating to volume, we arrive at energy per unit volume:

$$U = \frac{\sigma_x^2}{2E_{long}} = \frac{1}{2} E_{long} \epsilon_x^2 = \frac{E_{long} \epsilon^2}{2d} = \frac{I}{V} = \frac{I}{M} d \quad (32)$$

where  $d$  is fiber density,  $M$  its mass and  $I$  the energy absorbed through stretch; in the case of refining this is the energy transferred by bar impact.

An aqueous solution of pulp is pumped to the refiner where fibers and flocs are morphologically modified by successive and cyclic impacts from the rotor and stator bars, thus featuring refining as high

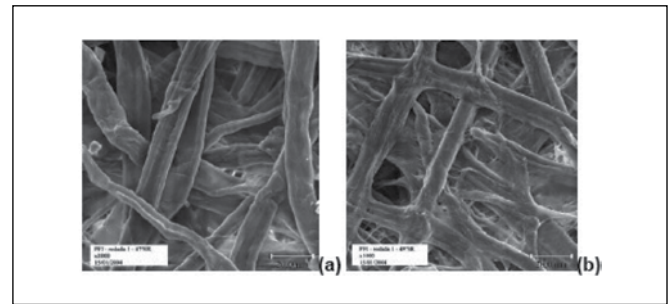


Figure 9. Bleached eucalyptus pulp fibers (a) before refining, (b) refined in PFI mill to 45°SR. From Yasumura (2008)

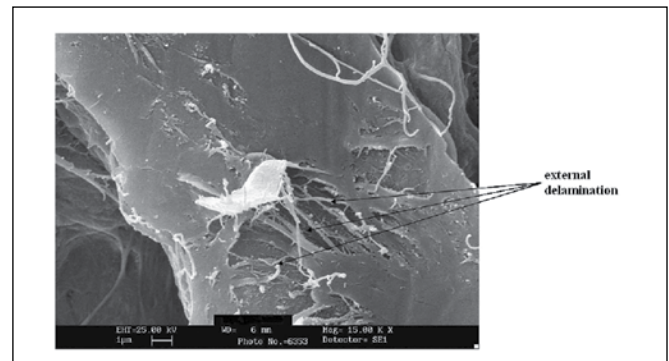


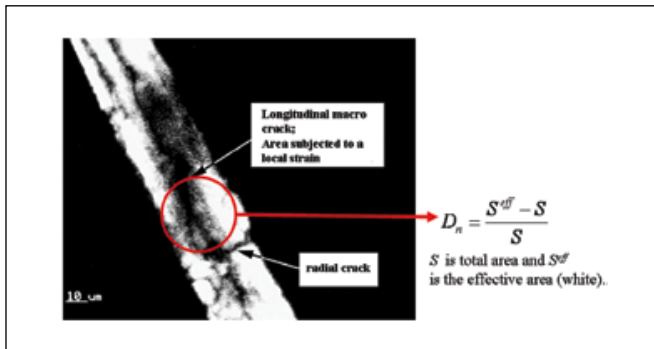
Figure 10. Micrograph of pulp fiber showing external delamination. From Parra (2003)

cyclic fatigue phenomenon. Depending on the intensity and number of impacts, the fiber may undergo external or internal fibrillation or even rupture. These effects are directly related to fiber fatigue. For instance: complete rupture of the fiber can be seen as the full fatigue of partial layers M, P, S1 or S2. External fibrillation may be thought as delamination of outer fiber layers due to tangential delamination, and internal fibrillation to between-layer delamination. **Figure 9** shows fibers before and after laboratory refining with a PFI mill. The reader can notice the appearance of the fibers which, on absorbing the energy of cyclic impacts of refining, has resulted in a geometric collapse. An experimental picture of external delamination due shear stresses can be seen in **Figure 10**.

Hamad (1997, 1998 and 2002) has been an active researcher in relating fatigue with pulp fiber refining. In a series of remarkable studies he has been investigating experimental methodology for fatigue testing of single fibers, and also the relatively new field of fractal simulation of crack propagation in fibers.

### DAMAGE VARIABLE APPLIED TO REFINING

Starting since the standard concept of damage (Lemaitre and Chaboche, 1990) from mechanics of solids, these ideas have been recently applied (Cuberos-Martinez, 2005) to low consistency refining in order to get a variable suitable to express the fiber fatigue and at the same time be a variable easy to handle in experimental works. This variable  $D$ , known as the damage variable, is defined as follows: consider a pulp fiber in which an element of finite volume has isolated, and let  $S$  be the area of a section of the volume element.



**Figure 11.** Representation of the defect variable. Taken and adapted from Hamad (1997)

This surface may contain cavities, scratches, cracks or other defects. The parameter  $S^{eff}$  is the effective area subject to tensions, so, by definition:

$$D_n = \frac{S^{eff} - S}{S}; S^{eff} \leq S \quad (33)$$

$D = 0$  corresponds to a defect-free sample and  $D = 1$  to a sample broken in half of the unit of volume, and  $0 < D_n < 1$  corresponds the damage state of the pulp fiber. **Figure 11** shows how this variable may be understood by using an example of a pulp fiber under cyclical stress.

In places where defects already exist, stress applied will become localized and concentrated, so stress may effectively be expressed by:

$$\sigma^{eff} = \frac{\sigma}{(1 - D)}, \quad (34)$$

with  $\sigma^{eff} = \sigma$  representing an idealized virgin pulp (without defects) and  $\sigma^{eff} \rightarrow \infty$  at the moment of fracture or rupture.

The generalized Hooke equation of damage can thus be written in its tensorial form:

$$\sigma_{ij} = (1 - D)C_{ijkl}\epsilon_{kl} \quad (35)$$

where  $C_{ijkl}$  is sometimes called stiffness tensor and  $\epsilon_{kl}$  is the strain.

The damage variable is not directly measurable. Quantitative measurement is attained by a selected variable that can perform fatigue. One possible and clever way is to correlate it with the Young's modulus of the material, and following it the Hooke equation would be rewritten as:

$$\sigma^{eff} = \frac{\sigma}{(1 - D)} = E\epsilon^{eff} \quad (36)$$

and

$$E^{eff} = E(1 - D) \quad (37)$$

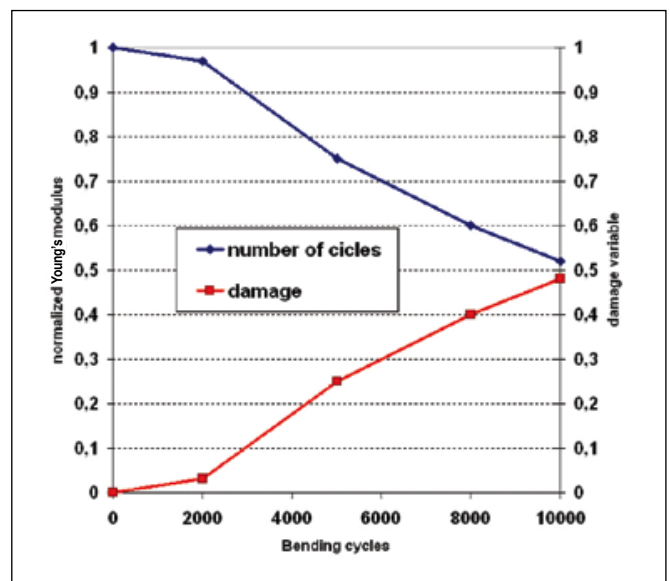
The term can be interpreted as the effective Young's modulus of refined fiber. Inverting last equation results in the expression of the damage variable:

$$D = 1 - \frac{\sigma}{E\epsilon^{eff}} = 1 - \frac{E^{eff}}{E} \quad (38)$$

In other words, reduction of the Young's modulus is proportional to the increase in the damage variable and this correlation can be seen in **Figure 12**, where pulp fibers were subjected to cyclic bending, simulating a refining operation.

This behaviour shows exactly what happens during refining: fibers bend more (less rigidity or lower Young's modulus) as they are under refining.

With respect to number of cycles, the damage variable follows a law of progression. In the refining case, a direct relation can be set up from analysis of some physical property of the paper obtained, according to the grade of refined pulp. One likely example is the breaking length, which is directly related to the tensile strength. This property gives, ideally, the length of a strip of paper that breaks under the action of the force of its own weight. Numerically, this property increases proportionally with refining until reaching a limit after which it begins to decline. This experimental behaviour is shown in Dasgupta (1994). This may be understood in the following way: in unrefined pulp – ideally considering a defect factor of zero or  $D = 0$  - individual fibers are rigid (high Young's modulus) with fewer links and low enmeshing capacity, thus forming few fiber-fiber bonds and, thereafter, the paper produced has lower stretch resistance – shorter tensile strength value – than paper made from refined pulp where fibers are more aligned and better externally fibrillated. At the limit at which mean damage value reach 1, the fiber will break itself, and virtually no fibrillation will happen, and the breaking length value numerically reverses as a consequence.



**Figure 12.** Correlation of defect variable with Young's modulus in pulp fibers subjected to cycle fatigue bending. Adapted from Tam Doo and Kerekes (1989)

## CONCLUSIONS

In the course of this study the physical nature of low consistency cellulosic pulp refining was reviewed. Some aspects from elasticity and fatigue theory and damage mechanics were used to express the effects of refining. A better understanding of microphysics behind refining is fundamental to model and project more efficient refiners. The introduction of damage variable is useful

tool to explain fatigue in low consistency refining, and searching for experimental and theoretical laws of damage evolution is in our point of view critical to understand the fatigue refining. Some concepts coming from composites materials engineering – like the fatigue growth of internal delamination under compressive loading – can be very useful when applied to pulp fibers, and remains as a subject of future work. ■

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